

## On the effective validation of charted soundings and depth curves

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### Abstract

Depth curves and charted soundings are two structural components of nautical charts, both derived from a more detailed dataset through generalization. Once depth curves are generated, the cartographer makes a selection of soundings that complements the depth curves and other features carrying bathymetric information in the adequate representation of the seabed morphology at the scale of the product. The selection of charted soundings – which is currently either performed fully manually, or partially manually using a computer-assisted solution – must meet the safety constraint, i.e., that no source sounding exists that is shoaler than what the mariner would expect by mentally interpolating the charted bathymetric information. According to International Hydrographic Organization S-4 publication, for well-surveyed areas that is achieved through the “triangular method of selection” and consists of two tests: the Triangle Test and the Edge Test. There are currently no fully automated solutions, so, with the ultimate goal of supporting “cartography at point of use”, we consider the problem of automating the validation of shoal-selected soundings. We discuss an implementation of the triangle test with improved performance near linear features, the first automated implementation of the edge test, and that the edge test may identify shoals that the triangle test fails to detect, confirming its significance in the validation process. We demonstrate an “intrinsic” limitation of the two tests that makes infeasible a fully automated solution based solely on the two tests. Finally, to overcome the intrinsic limitation, we propose a new validation test, named Nautical Surface Test, that captures the local morphology at the appropriate charting resolution as the solution for the automated validation of the charted bathymetric information.

### Introduction

The compilation of bathymetry on nautical charts is one of the most critical and time-consuming processes in nautical cartography. Charte depth curves and soundings are derived from more detailed (source) datasets, either survey data and/or larger scale charts, with cartographic generalization. The process is a continuous compromise among the chart legibility, topology, morphology, and safety constraints as they are often incompatible with each other [1]. Once depth curves are created, the cartographer, following established cartographic practice rules (see, e.g., [2], [3]), makes the selection of the soundings that will be charted. The selection (as well as the depth curves’ compilation) is performed either fully manually and/or with using one of the existing software solutions. For manual selection, the cartographer first selects the least depths, critical, controlling, and supporting soundings, and subsequently the other soundings necessary for the representation of the seabed morphology on the chart. When a chart already

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exists in the area, the cartographer uses the distribution of soundings on the existing chart as a guiding subset for the selection of the additional soundings. The initial selection is then evaluated and corrected where necessary to meet the prominent constraint of safety, i.e., that the expected water depth based on the charted bathymetric information should not appear, at any location, deeper than the source information. Figure 1 illustrates the process for the compilation of bathymetry from the source information to its application to the chart.

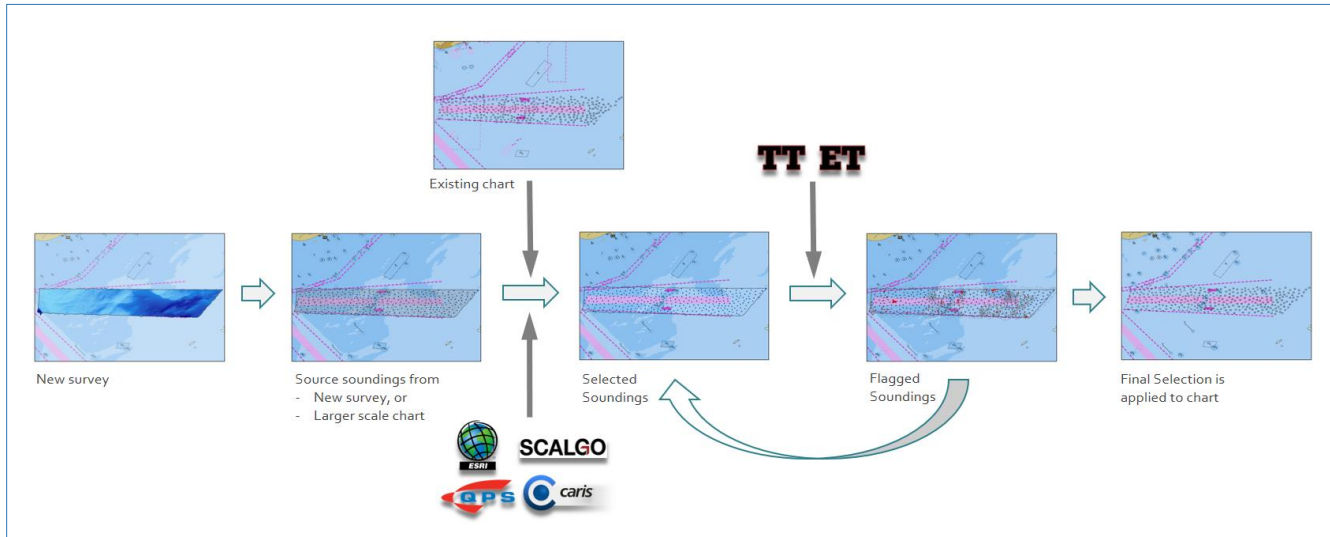


FIGURE 1. From source information to charted soundings.

According to the International Hydrographic Organization’s (IHO) S-4 Chart Specifications [2], the “shoal-biased pattern” of selection for the charted soundings is achieved through the “triangular method of selection”, and more specifically through two tests, i.e., the *Triangle* and *Edge Tests* (TT and ET in Figure 1). For the triangle test the cartographer is called to verify that no actual sounding (hereinafter: source sounding) exists within a triangle of selected soundings which is shoaler than the least depth of the soundings forming the triangle. Likewise, for the edge test, no source sounding may exist between two adjacent selected soundings shoaler than the least of the two selected soundings forming an edge of the triangle.

Chart compilation follows strict rules and established cartographic practices described in national and international standards, but, as with any other cartographic product, the outcome varies according to cartographer’s perception and subjective decisions. It is arguable – and an open research question in nautical cartography – whether we can fully substitute the human component in the compilation process. However, for many of the compilation tasks, automation may ensure objectivity and consistency, elimination of human errors, as well as increased productivity and reduced chart production and updating times.

In the past few decades researchers and software vendors have developed several automated algorithms, most of them focusing on the simplification of isobaths and the soundings selection (e.g., [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]). Concerning the validation of the shoal-biased pattern of the selected soundings, the first implementation effort of the triangle test was introduced in the context of HydrOffice [16]. HydrOffice is an open research framework for ocean mapping, jointly developed by the

National Oceanic and Atmospheric Administration/Office of Coast Survey (NOAA/OCS) and the University of New Hampshire/Center for Coastal and Ocean Mapping (UNH/CCOM) ([17], [18]). The automation of the triangle test in QC Tools was a significant contribution to tackling the time consuming and error-prone task of validation. The specific implementation, which followed the IHO S-4 concept for triangles formed by selected soundings, performed well in open areas on the chart (i.e., areas away from depth curves and coastlines) but, generally, failed in validating the bathymetric information in vicinity to linear bathymetric features. Although the situation was improved in later versions of the tool (e.g., by incorporating selected nodes from depth curves in the generation of a constrained TIN), additional research toward more robust algorithms was required.

In this work, we present the results of a developing research project on the automated validation of the charted bathymetric information. That includes an implementation of the triangle test with improved performance in areas near and within linear bathymetric features and the areas adjacent to the boundaries of the new survey area, as well as the first automated implementation of the edge test described in the literature. The above implementations, besides the selected soundings, incorporate the available bathymetric information on charts in the form of points and lines (e.g., rocks, depth curves, and coastlines) for the generation of a conforming Delaunay triangulation. Furthermore, we present an intrinsic limitation of the above two tests that makes impossible the materialization of a fully automated solution based on the two tests only, and a new validation test, called Nautical Surface Test, “surface-test” for short, that identifies discrepancies that both the triangle and edge tests fail to identify.

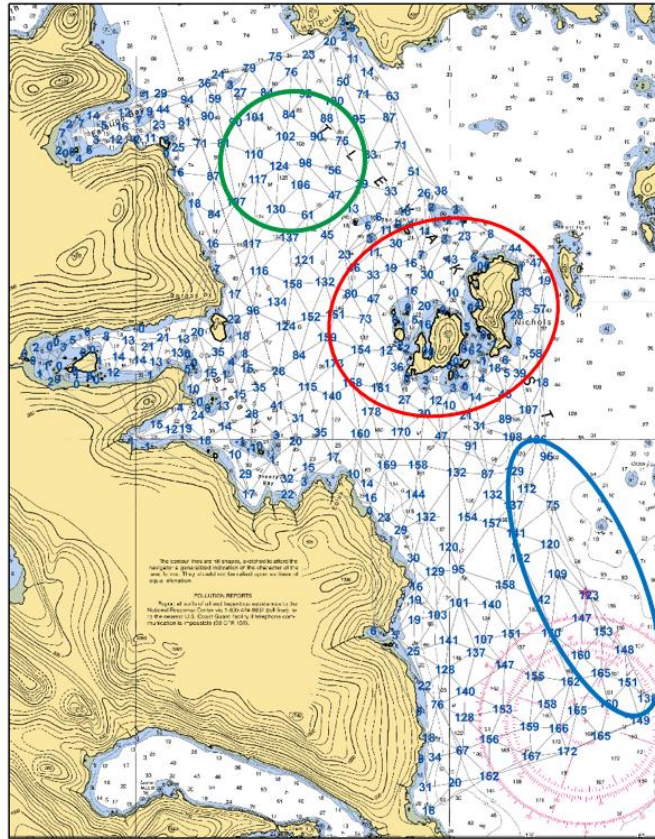
## Triangle and Edge Tests

The aim of this research project has been the automated validation of the shoal-biased pattern of the charted bathymetric information. The research work initially focused on the two tests described in the IHO S-4 publication, i.e., the triangle and edge tests. The available implementation of the triangle test in HydrOffice QC Tools shed light to areas subject to improvements, i.e., those near and within linear features, and near the boundaries of the new area, where the existing tools are not effective. To improve that situation, we worked on identifying cartographers’ approach to the validation problem and, to the extent possible, to algorithmically replicate it.

In that context, we identified three geographic areas that the validation is handled differently, i.e.:

1. Open areas, meaning areas that the bathymetry and the characteristic features of the sea floor are represented on the chart only with point depth features (e.g., selected soundings, rocks), e.g., the area marked green in Figure 2. In these areas, the validation is performed with searching within triangles and along edges formed by the point features for any discrepancies.
2. Areas near and within linear features carrying bathymetric information (e.g., depth curves, coastlines, channel frameworks), e.g., the area marked red in Figure 2. There, the cartographer performs the validation using the areas of dominance of the charted features. Between two linear features, the cartographer examines the source soundings that lie in the area between the two lines for any discrepancies. Between a sounding and a linear feature, the cartographer performs the validation for the source soundings within the combined area of dominance of the point and the linear feature.
3. Near the boundaries of the new area (e.g., area marked blue in Figure 2). For the validation, the cartographer mentally incorporates the charted bathymetric information from the adjoining areas

and compares the source information against the combined datasets (following the above two approaches).

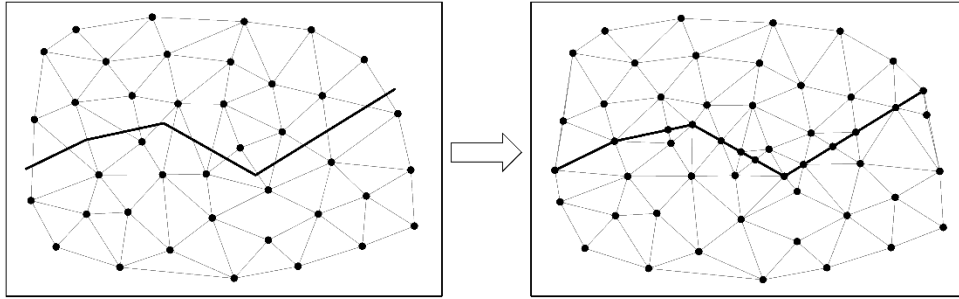


**FIGURE 2.** Examples of an open area (marked in green), an area near and within depth curves and coastlines (marked in red), and areas near the boundaries of the new area (in blue), each being treated differently by the cartographer.

One important conclusion from the above is that the selected soundings must be validated in conjunction with the other bathymetric features on the chart. The selection of soundings aims to complement other features carrying bathymetric information in the representation of the submarine relief on charts. Inclusion of one may result to the exclusion of another and vice versa. Therefore, a fully automated solution must incorporate the entirety of the charted bathymetric information, not a selection of it. Furthermore, to account for the areas near the boundaries of the new survey, the charted bathymetric information from the adjoining areas must also be incorporated. That is achieved by buffering the newly compiled area, and selecting and bringing in the charted bathymetric information that lies within the buffer.

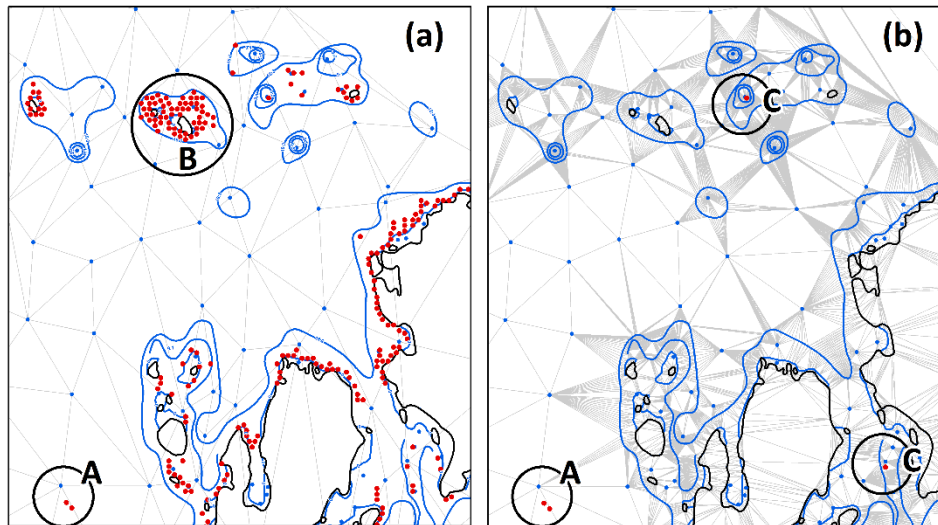
For the above bathymetric features, we generate the conforming Delaunay triangulation which is then used for the validation utilizing the mandates for the triangle and edge tests. The selection of the conforming over the ordinary Delaunay triangulation ensures that the resulting Delaunay edges will not cross any linear bathymetric features (Figure 3), something that would, otherwise, yield many false positives and make the validation near linear features problematic. It is noted that the areas of dominance that were previously described for the validation near linear features correspond to Voronoi polygons [19], but for the purpose of the validation task they may be sufficiently examined with the triangles

generated with the conforming Delaunay triangulation. The Delaunay triangulation and Voronoi diagram are commonly used in geosciences for the study and description of a large variety of phenomena (see, e.g., [1], [20], [21], [22], [23], [24], [25], [26], [27], [28]).



**FIGURE 3.** Ordinary Delaunay triangulation for a set of points (left) and the conforming Delaunay triangulation for the same set of points and a linear feature (right). [29]

Figure 4 compares the performance of the proposed implementation for the triangle test (hereinafter: “proposed implementation”) to an implementation that constructed the TIN using only the selected soundings (hereinafter: “other implementation”). The other implementation follows a verbatim interpretation of the triangle test, as written in S-4, according to which “no actual sounding exists within a triangle of selected soundings”, thus, not considering the linear features in the area. It is obvious that in open areas and away from linear features both implementations successfully identify shoal soundings within triangles formed by selected soundings (e.g., the two flags marked with “A” in the south-western side of Figure 4). However, near linear features the other implementation (Figure 4a) performs poorly as it returns an enormous number of false positives (e.g., area “B” in Figure 4a), contrary to the proposed implementation (Figure 4b) which flagged only the actual shoals in these areas (“C” in Figure 4b).

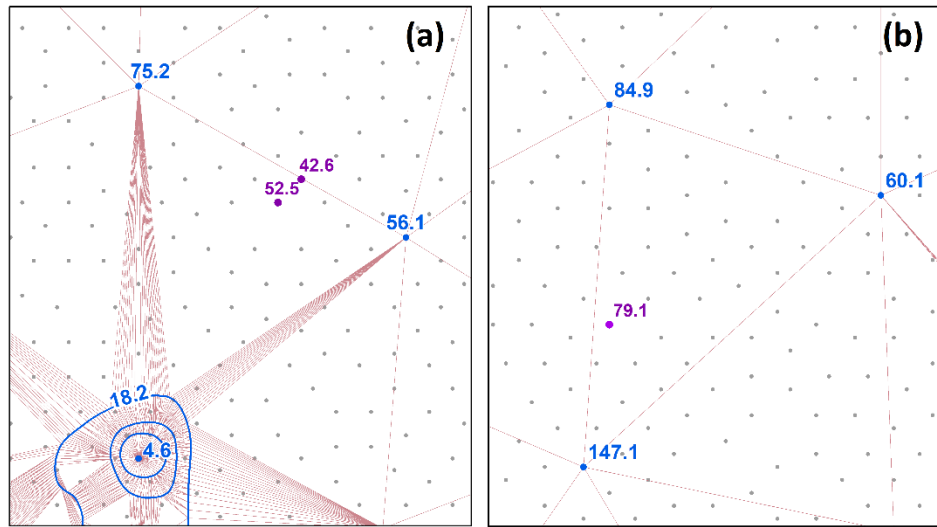


**FIGURE 4.** (a) The triangle test using only the selected soundings for the construction of the TIN, and (b) the proposed implementation which incorporates all the available bathymetric information from the selected soundings, depth curves, and coastlines. [29]

Following those described previously for the generation of a conforming Delaunay triangulation, we perform the second test described in IHO S-4, i.e., the edge test, searching along the triangles’ edges for



discrepancies. Currently a buffer of 10 percent the edge's length is used (determined with the trial-and-error-method), but it constitutes part of our future work to determine the best buffer size and shape (see [29]). Figure 5 illustrates the importance of the edge test in the validation process showing two geographic areas with three shoals that the triangle test failed to identify. More precisely, in Figure 5a the soundings 42.6m and 52.5m are flagged with the edge test as they are shoaler than the two selected soundings forming the edge (i.e., soundings 56.1m and 75.2m). In terms of the triangle test, the soundings in question are deeper than the adjacent 18.2m depth curve, a vertex of which forms the local triangle, and, as such, are not shoals. Likewise, in Figure 5b the edge test flagged the sounding 79.1m that deviates significantly from the expected depth in the area but was not detected by the triangle test due to the third selected sounding forming the local triangle (60.1m). Our implementation of the edge test is the first known implementation effort in the literature. Although the edge test had been ignored by existing implementation efforts, Figure 5 demonstrated its importance in the validation process as it may identify shoals that the triangle test fails to identify. Relying solely on the triangle test is therefore inadvisable.

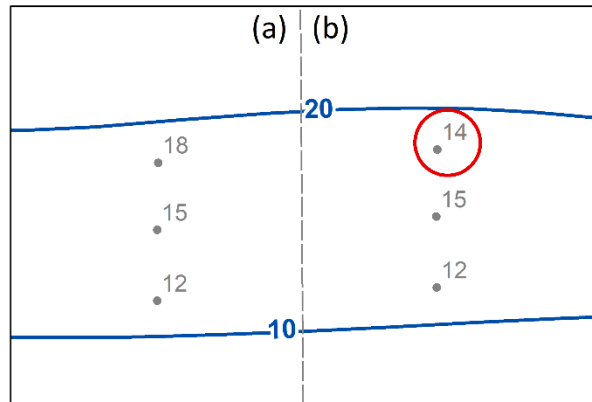


**FIGURE 5.** The significance of the edge test in identifying shoals that the triangle test, by definition, may not detect. [29]

## The Intrinsic Limitation

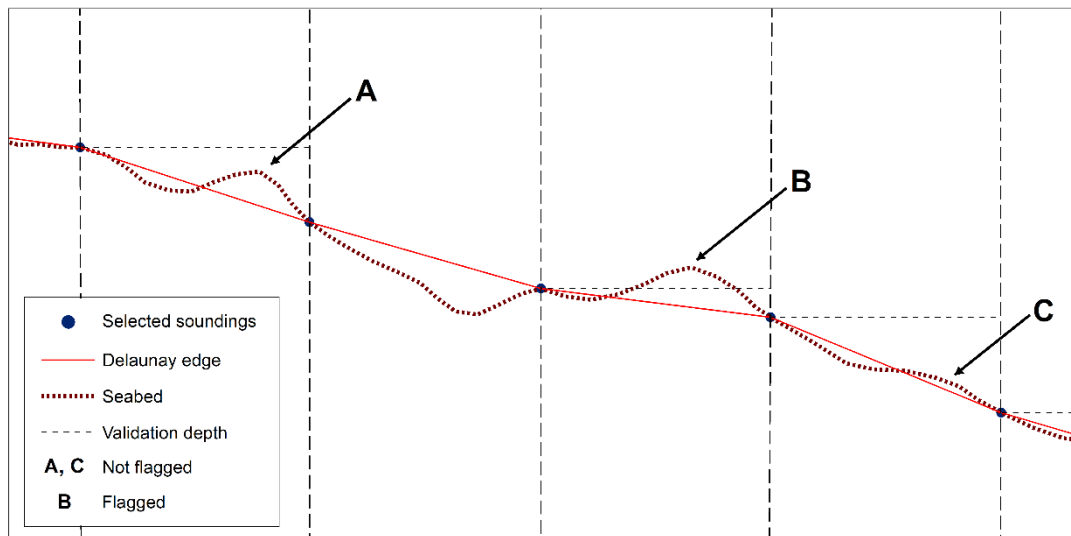
The research work documented the individual limitations of the triangle and edge tests, and revealed a fundamental limitation of the two tests that prevents the materialization of a fully automated solution based solely on them. We consider the fundamental limitation as “intrinsic” because it is the result of the definition of the two tests as described in the IHO S-4 publication, thus independent of any algorithmic implementation effort (see [29]). To illustrate this issue, Figure 6 presents two depth curves (10m and 20m) and source soundings between the two. On the left side of the dividing line (Figure 6a), the values of the source soundings follow a distribution that one would expect between the portrayed depth curves. On the right side of the dividing line (Figure 6b), the 14m sounding is approximately 20 percent shoaler than the expected depth at the specific location (based on the configuration of the two curves), and, as such, should be brought to the cartographer's attention for evaluation. However, the two tests fail to detect the specific discrepancy because (according to the tests' definition) the source information is compared to the least of the two or three vertices forming an edge or triangle. In this specific example,

for all vertices forming triangles and edges from the depth curves 10m and 20m, the comparison depth is 10m.



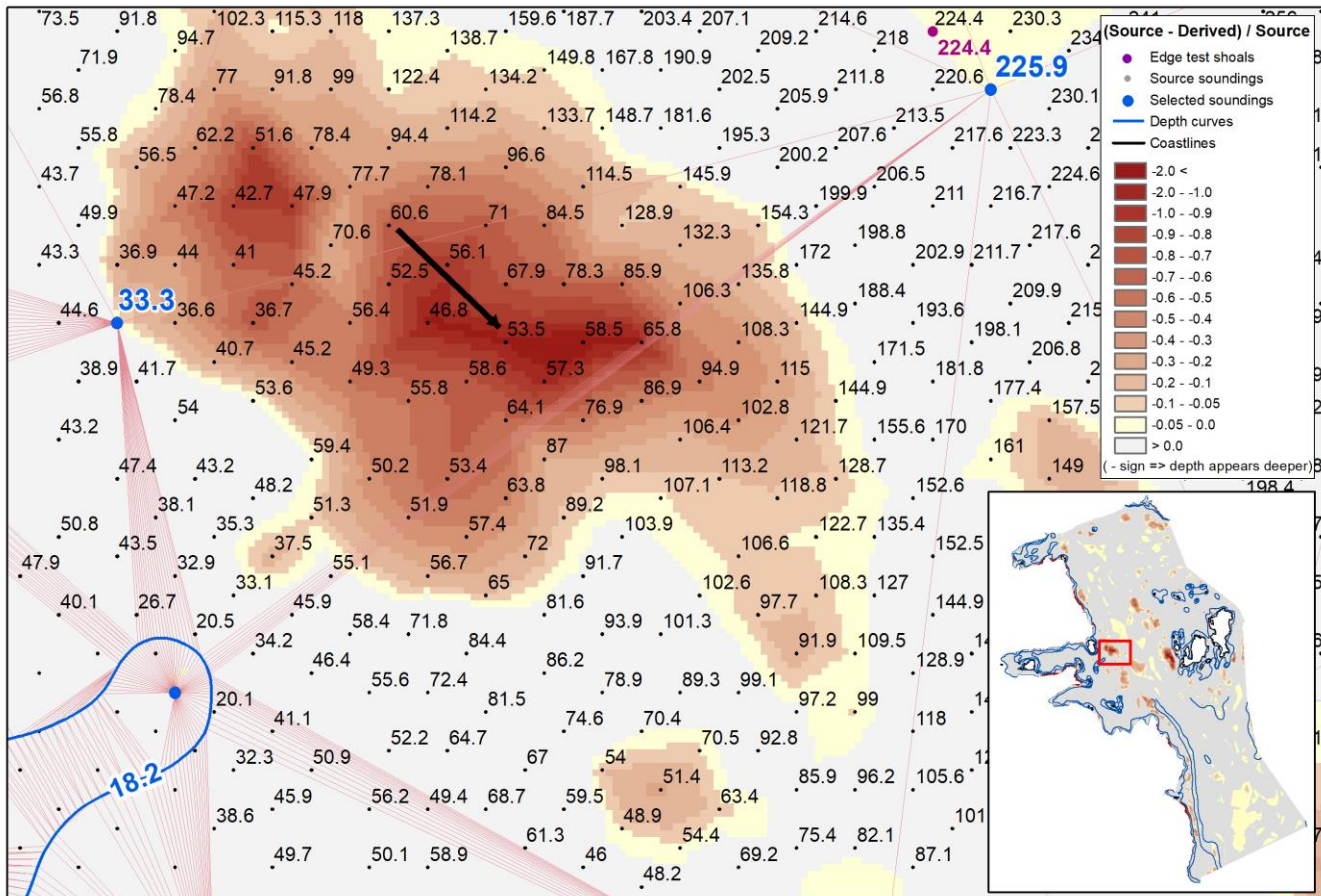
**FIGURE 6.** Source sounding (14m) that deviates significantly from the expected depth but the triangle and edge tests fail to identify due to their intrinsic limitation. [29]

In practice, the two tests generate a rough approximation of the surface represented by the charted bathymetric information using a gridding approach with an enormously big element. Each element is assigned the depth value of the shoalest of the two or three vertices forming the edge or triangle respectively and is compared to all source soundings within the specific element for the validation process. To illustrate this, Figure 7 presents a profile view of the seabed based on the available source information (brown dotted line in Figure 7) and the Delaunay faces (red lines in Figure 7) generated from the selected soundings (blue points in Figure 7). The horizontal dashed lines represent the vertical section of the discussed elements that serve as the validation depth for identifying the areas violating the safety constraint. With this approach, the eminences crossing the horizontal dashed lines are flagged (e.g., shoal “B” in Figure 7), but anything below the validation depth is not (“A” and “C” in Figure 7), even if it deviates significantly from the expected depth in the area (shoal marked “A” in Figure 7).



**FIGURE 7.** A profile view of the seabed, the selected soundings, and the Delaunay faces showing why the two tests fail to identify eminences that deviate significantly from the expected depth on chart. [29]

Figure 8 presents a real-world example of the fundamental “intrinsic” limitation of the two tests. For the presented area, the source soundings satisfy both the triangle and edge tests (with the exception of the insignificant 224.4m sounding in purple color flagged with the edge test) albeit the charted information fails to maintain and emphasize the morphological details of the seafloor. The underlying raster represents the difference between the actual source surface and the surface derived from the charted information with linear interpolation. Characteristically, at the location of the pointing arrow the interpolated depth appears twice deeper than the actual depth (109m instead of 53.5m).



**FIGURE 8.** Area where the source soundings satisfy both tests, but the interpolated surface appears twice deep as the actual source surface, a clear violation of the safety constraint.

Figure 8 demonstrated the negative effect of the intrinsic limitation of the two tests to the representation of the sea-bottom topography on charts. Although the specific example does not pose a threat to surface ship navigation, it is essential to acknowledge that corresponding situations may appear in any depth range (some of which may indeed endanger safe navigation), thus undermining the effort that the hydrographic community puts in the collection, processing, and portrayal of bathymetry on charts.

Besides the effect of the above limitations to chart production, a similar problem exists in the identification of chart discrepancies after new survey data. In an effort to solve the problem in the hydrographic realm the, so-called, Tilted-Triangle test was proposed in [30] and made publicly available



as part of the HydrOffice's Chart Adequacy (CA) Tools [31] (Figure 9). However, for chart production and the automated validation of charted bathymetric information, further research work was required.

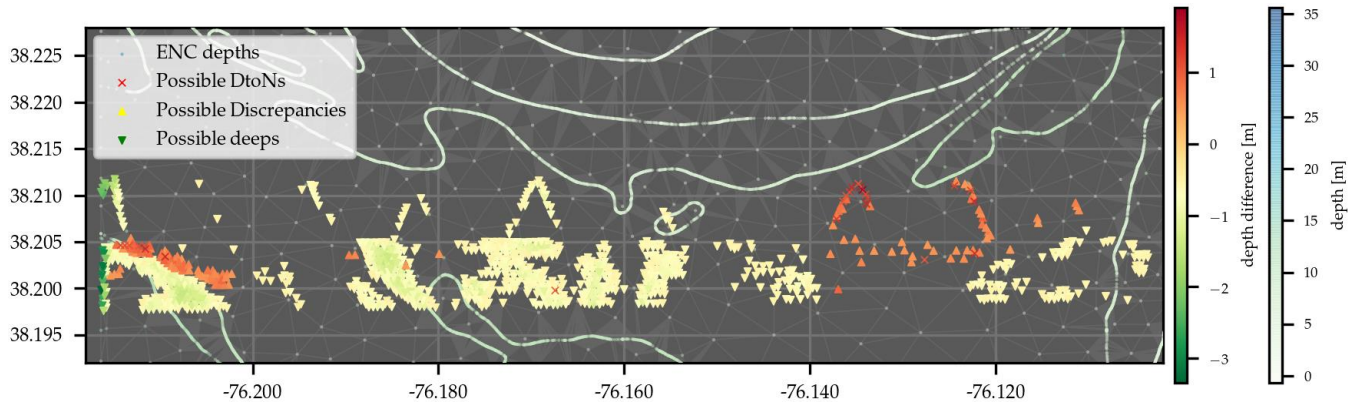


FIGURE 9. Example of the results of the Tilted Triangle test in HydrOffice CA Tools. [30]

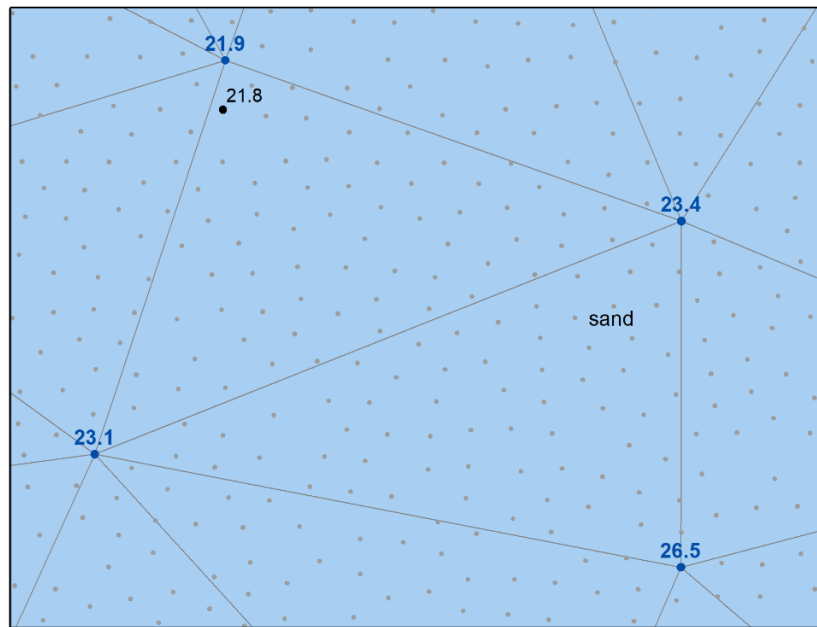
## Nautical Surface Test

The intrinsic limitation of the triangle and edge tests, presented with the support of Figures 6-8, made evident that a fully automated process for the validation of the shoal-biased pattern of the charted bathymetric information based solely on the two tests is not feasible. Such an automated solution cannot spot discrepancies that the experienced cartographer may detect. Therefore, this research work investigated the development of a new test that would complement or supersede the two tests toward a fully automated solution. The new test should account at the appropriate charting resolution for the configuration of the seabed and capture the relevant discrepancies between the source and the selected bathymetric information for charting.

For the triangle test and the edge test, the source information is compared against a distant depth value which happens to be the shoalest of the two or three depth vertices forming an edge or triangle, causing the problems presented in the previous section. To overcome the intrinsic limitation of the two tests, we propose a new test, named *Nautical Surface Test* (NST), “surface-test” (ST) for short, for which the source soundings are compared to the “expected” depth at the exact location of the source soundings. For each source sounding, the proposed test interpolates the charted bathymetric information and compares the calculated value to the depth value of the source sounding. If the former is greater (meaning that the depth at this location appears deeper than the measured depth), the source sounding is flagged. There are several interpolation methods described in the literature (e.g., Linear Interpolation, Natural Neighbors, Inverse Distance Weighting, Kriging, Spline) that may be used with the described test, each of them with advantages and disadvantages (see, e.g., [32], [33]). Currently, our implementations of the surface test incorporate the Delaunay triangulation with Linear Interpolation (“NST-L”) and the Natural Neighbors (“NST-N”), but which interpolation method performs best for the bathymetric information on charts (including how bathymetry is perceived by mariners) is an open research question.

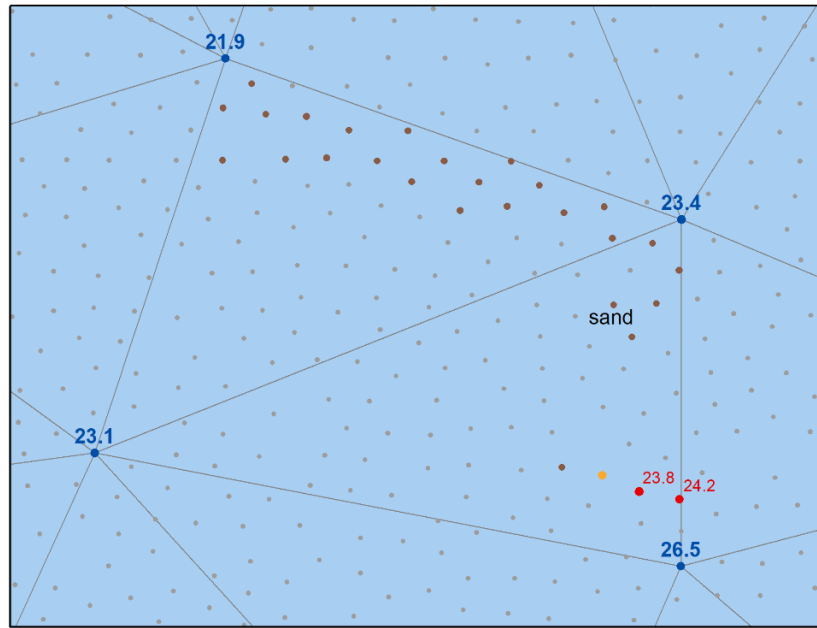
The following Figures 10-12 present the results of the surface test (and more precisely the NST-L) and the two traditional tests (triangle and edge tests) in two geographic situations, demonstrating the superiority of the new test. In these figures, the selected soundings appear in blue color, the source soundings in light grey, the soundings flagged with the traditional tests in black and those flagged with the NST-L in dark

grey. In Figure 10, for the traditional two tests all source soundings within the northern triangle are compared to the selected sounding 21.9m and those in the southern triangle are compared to the 23.1m selected sounding. The two tests identified as shoal only the 21.8m source sounding portrayed in black color. Clearly, 0.1m difference at the location of the 21.8m is insignificant compared to the 21.9m selected sounding and, thus, the 21.8m flag may be ignored. Based on the above, one may draw the conclusion that the current selection of soundings honors the safety constraint and that the area in question passes the validation test.



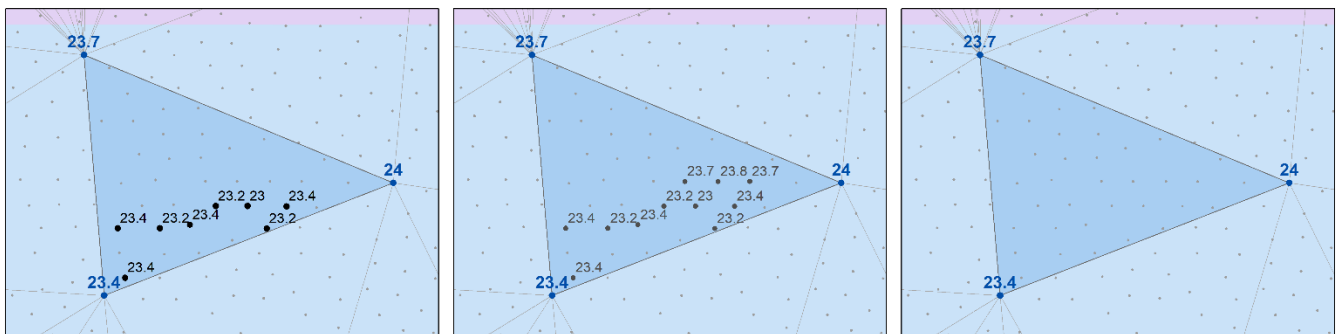
**FIGURE 10.** With only the 21.8m source sounding flagged with the triangle and edge tests, one may conclude that the area enclosed by the two triangles meets the safety constraint.

On the contrary, as Figure 11 illustrates with utilizing the results from the surface test, the current selection of soundings in the area is subject to improvements. In detail, the source soundings emphasized in Figure 11 (i.e., soundings portrayed in dark grey, orange, and red color) are source soundings that are shoaler than the interpolated depth at the specific location and, consequently, flagged by the surface test. Similarly to the 21.8m sounding in Figure 10, the flagged soundings in Figure 11 may also be insignificant. One of the advantages of the surface test is that a tolerance may be applied to the identified shoals. It seems reasonable that the tolerance (i.e., the maximum value that the calculated depth may be deeper than the source sounding) for the surface test shall derive from the populated Category Zones of Confidence (CATZOC) [34] value in the area. The specific area of Figures 10 and 11 has been assigned CATZOC A1, meaning that the acceptable depth accuracy is  $0.5\text{m} + \text{one percent of the depth}$  (roughly, 75cm). By applying the tolerance to the above results, all the soundings in dark grey are within the acceptable A1 limits and, as such, ignored. On the contrary, at the location of the sounding in orange the expected depth exceeds the allowable limit of depth accuracy for A1 and falls within CATZOC A2 limits (up to  $1\text{m} + \text{two percent of the depth}$ ), and, worse, at the location of the two soundings in red exceeds the A2 limits and falls within CATZOC C (up to  $2\text{m} + \text{five percent of the depth}$ ).



**FIGURE 11.** The surface test identified multiple locations where the expected depth is deeper than the source information. Those in dark grey fall within CATZOC A1 limits and may be ignored, that in orange falls within A2, and, worse, the two in red are within CATZOC C.

The utilization of depth tolerance, as shown in the previous example, helps to distinguish the significant from insignificant flags and it is an important advantage of the surface test over the triangle and edge tests. The use of a tolerance value with the traditional two tests would make them behave unpredictably. To elaborate, with applying, e.g., 0.1m tolerance to the example presented in Figure 10, the 21.8m insignificant shoal is successfully removed from the exported results. However, if the 14m sounding in Figure 6 was 9.9m, with the 0.1m tolerance the 9.9m would also be removed although it would constitute a significant shoal and potential danger to navigation. Figure 12 illustrates a characteristic example of a situation where the triangle and edge tests flagged eight source soundings (Figure 12a) and the NST-L flagged 12 source soundings (including the eight flagged with the two tests) (Figure 12b) in the triangle formed by the three selected soundings 23.4m, 23.7m, and 24m. The eight soundings flagged with the two tests need to be inspected by the cartographer, whereas with the surface test and the applied tolerance all 11 shoals are automatically removed (Figure 12c).



**FIGURE 12.** (a) The soundings flagged with the triangle and edge tests, (b) Soundings flagged with the surface test, (c) The results of the surface test after the CATZOC A1 tolerance is applied.

Removing insignificant shoals from the exported results helps the cartographer focus on the most significant shoals. The triangle test and the edge test often result in an enormous number of insignificant shoals, but as explained previously, it is unsafe to apply any form of tolerance to the two tests. The surface test, in conjunction with an applied tolerance value, helps cartographers by removing the less significant shoals, thus preventing an error-prone “cluttered” situation where the cartographer may fail to identify the significant shoals and properly improve the soundings selection.

The research effort presented in this paper led to the implementation of a toolset for the validation of the shoal-biased pattern of the charted bathymetric information that is becoming operational with NOAA/OCS Marine Chart Division. The foundation of the automated solution is the surface test but, if necessary by the specific geographic configuration (e.g., within channel frameworks) or the interpolation method used (e.g., the case of flat triangles when Delaunay with Linear Interpolation is used), a point-in-polygon test, where the source soundings are validated against the underlying depth area, is incorporated.

## Conclusions

This paper presented our research work toward the automated validation of the shoal-biased pattern of the charted bathymetric information. It presented the improved performance of an implementation of the IHO S-4 triangle test that incorporates the available bathymetric information within the new area and the adjoining charted areas for the generation of a conforming Delaunay triangulation, and the first known implementation effort of the Edge Test together with its significance in the validation process. It also presented the, by definition, “intrinsic” limitation of the two tests that prevents a fully automated solution based on the triangle and edge tests. Finally, this paper described a new test, named Nautical Surface Test (“surface-test” for short), that overcomes the intrinsic limitation and that is able to detect discrepancies that the two tests fail to identify. The innovative surface test in conjunction with a tolerance value disregards the insignificant shoals, thus preventing a cluttered situation of significant and insignificant shoals that may obscure cartographer’s focus on distilling the data to the most relevant information for the mariner.

## Funding

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